

Control of a Delayed Inverted Pendulum*

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Abstract

This paper describes a visual feedback control scheme for an inverted pendulum, using a vision system equipped with a digital camera as sensor to obtain information of the system. Results are presented in simulation.

1 Introduction

Traditionally visual sensing and manipulation are combined in an open-loop fashion, 'looking' then 'moving'. The accuracy of the operation depends directly on the accuracy of the visual sensor and the manipulator and its controller. An alternative to increasing the accuracy of these subsystems is to use a visual feedback control loop, which will increase the overall accuracy of the system: a principle concern in any application. The term visual servoing was introduced by Hill and Park in 1979 to distinguish their approach from earlier 'blocks world' experiments where the system alternated between picture taking and moving [3].

Visual feedback is the use of the visual information in control, using elementary areas including digital signal processing, kinetics, dynamics, control theory, real-time computing among others [3, 5, 6].

Underactuated mechanical systems are systems with fewer actuators than degrees of freedom.

Exist many examples, such as the inverted pendulum, the gymnast robots and particularly the acrobat, the pendubot, the planar vertical takeoff and landing aircrafts, the undersea vehicles and other mobile robots [1, 2, 8].

In the literature many research efforts have been made on control aspects but the field of control of such systems still open to develop other control strategies.

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The remainder of the paper is organized as follows: In Section 2 the modeling of the inverted pendulum system and the camera of vision is presented. The controller design is obtained in Section 3. Section 4 presents the simulation results. Finally, the conclusions and future works are given in the Section 5.

2 Modeling of the Camera and Inverted Pendulum System

In this section, we develop a mathematical model for the inverted pendulum as well as camera and study the properties of these models.

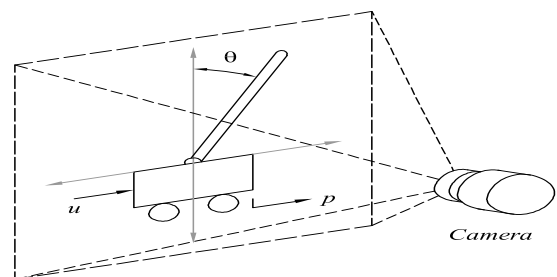


Figure 1: Camera inverted pendulum system.

2.1 Inverted Pendulum System

An inverted pendulum is a physical device that consists on one cylindrical bar (pendulum) that oscillates freely around a fixed point (with certain mechanical restrictions, since just it can move in a plane). This pendulum is mounted on a mobile piece (cart) that moves in horizontal direction [9]. The objective of this paper is the control with visual feedback of the inverted pendulum where the sensor is a digital camera to measure the position of the Pendulum and the cart. The Figure 1 shows the camera

inverted pendulum system, whose main parameters are given by the Table 1.

Table 1: Parameters and the values of the system.

Var.	Name	Values	Units
u	Input		<i>volts</i>
p	Car position		<i>m</i>
θ	Angle between the vertical and the position of the pendulum		<i>rad</i>
l	Length to the center of mass of the pendulum	0.32	<i>m</i>
m	Pendulum mass	0.23	<i>kg</i>
M	Car mass	0.52	<i>kg</i>
g	Gravity acceleration	9.81	<i>m/sec²</i>
J	Pendulum moment of inertia, about its center of gravity	0.007	<i>Kg · m²</i>

The corresponding equations of motion are derived using Lagrange's equations [11]

$$\frac{d}{dt} \left[\frac{\partial L}{\partial \dot{\mathbf{q}}} \right] - \frac{\partial L}{\partial \mathbf{q}} = \boldsymbol{\tau} \quad (1)$$

where L denotes the Lagrange-function, defined by $L = K - V$, where K is the kinetic energy and V is the potential energy.

For the inverted pendulum the kinetic energy is given by

$$K = \frac{1}{2} M \dot{p}^2 + \frac{1}{2} m v^2 + \frac{1}{2} J \dot{\theta}^2 \quad (2)$$

where

$$v^2 = \dot{p}^2 + l^2 \dot{\theta}^2 + 2 \dot{p} \dot{\theta} l \cos \theta$$

while the potential energy, is given by

$$V = mgl \cos \theta \quad (3)$$

where the generalized coordinates vector is

$$\mathbf{q}(t) = \begin{bmatrix} p(t) \\ \theta(t) \end{bmatrix} \quad (4)$$

resulting the nonlinear equations that describe the system dynamics

$$(M+m) \ddot{p} + ml \ddot{\theta} \cos \theta - ml \dot{\theta}^2 \sin \theta = u \quad (5)$$

$$(ml^2 + J) \ddot{\theta} + ml \ddot{p} \cos \theta - mgl \sin \theta = 0 \quad (6)$$

choosing the state variables as

$$x_1 = p, \quad x_2 = \dot{p}, \quad x_3 = \theta, \quad x_4 = \dot{\theta} \quad (7)$$

we have

$$\dot{x}_1 = x_2 \quad (8)$$

$$\dot{x}_2 = \frac{-m^2 l^2 g \sin x_3 \cos x_3 + (ml^2 + J)(mlx_4^2 \sin x_3 + u)}{(M+m)(ml^2 + J) - m^2 l^2 \cos^2 x_3} \quad (9)$$

$$\dot{x}_3 = x_4 \quad (10)$$

$$\dot{x}_4 = \frac{mgl(M+m) \sin x_3 - ml(u + mlx_4^2 \sin x_3) \cos x_3}{(M+m)(ml^2 + J) - m^2 l^2 \cos^2 x_3} \quad (11)$$

in matrix form (8)-(11)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x}) u \quad (12)$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_2 \\ \frac{-m^2 l^2 g \sin x_3 \cos x_3 + (ml^2 + J)(mlx_4^2 \sin x_3)}{(M+m)(ml^2 + J) - m^2 l^2 \cos^2 x_3} \\ x_4 \\ \frac{mgl(M+m) \sin x_3 - ml^2 x_4^2 \sin x_3 \cos x_3}{(M+m)(ml^2 + J) - m^2 l^2 \cos^2 x_3} \end{bmatrix} \quad (13)$$

and

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 \\ \frac{ml^2 + J}{(M+m)(ml^2 + J) - m^2 l^2 \cos^2 x_3} \\ 0 \\ -\frac{ml \cos x_3}{(M+m)(ml^2 + J) - m^2 l^2 \cos^2 x_3} \end{bmatrix} \quad (14)$$

The proposed outputs are $y_1(T) = p(t)$ and $y_2(T) = \theta(t)$.

2.1.1 Linear model

For the design of the controller the model is linearized around an equilibrium point $\mathbf{x}^* = \mathbf{0}$ by the Taylor's series expansion of (12), we have a linear model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (15)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad (16)$$

where

$$\mathbf{A} = \frac{\partial f}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}^*} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-m^2 l^2 g}{a} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{mlg(M+m)}{a} & 0 \end{bmatrix} \quad (17)$$

$$\mathbf{B} = \mathbf{g}(\mathbf{x}^*) = \begin{bmatrix} 0 \\ \frac{ml^2 + J}{a} \\ 0 \\ \frac{-ml}{a} \end{bmatrix} \quad (18)$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (19)$$

where

$$a = (M + m)(ml^2 + J) - m^2 l^2.$$

2.2 Discrete Model

In this section we obtain a discrete linear model that it will be used for the control design. Using a sampling period $T = 5 \text{ ms}$, the discret system obtained is the following

$$\mathbf{x}((k+1)T) = \mathbf{\Phi} \mathbf{x}(kT) + \mathbf{\Gamma} u(kT) \quad (20)$$

$$\mathbf{y}(kT) = \mathbf{C} \mathbf{x}(kT) \quad (21)$$

where

$$\mathbf{\Phi} = \begin{bmatrix} 1 & 0.005 & 0 & 0 \\ 0 & 1 & -0.0152 & 0 \\ 0 & 0 & 1.0004 & 0.005 \\ 0 & 0 & 0.1546 & 1.0004 \end{bmatrix} \quad (22)$$

$$\mathbf{\Gamma} = \begin{bmatrix} 0 \\ 0.0087 \\ 0 \\ -0.0210 \end{bmatrix}. \quad (23)$$

2.2.1 Controlability and Observability

The necessary and sufficient condition for the system to be controllable (observable) is: the controlability (observability) matrix (24) y (25) must be full rank [7].

$$\mathbf{W}_c = [\mathbf{\Gamma} \quad \mathbf{\Phi} \mathbf{\Gamma} \quad \mathbf{\Phi}^2 \mathbf{\Gamma} \quad \mathbf{\Phi}^3 \mathbf{\Gamma}] \quad (24)$$

$$\mathbf{W}_o = [\mathbf{C} \quad \mathbf{C} \mathbf{\Phi} \quad \mathbf{C} \mathbf{\Phi}^2 \quad \mathbf{C} \mathbf{\Phi}^3]^T \quad (25)$$

2.3 Camera Model

Here we propose a camera model. It is consider as delay system with a proportional gain.

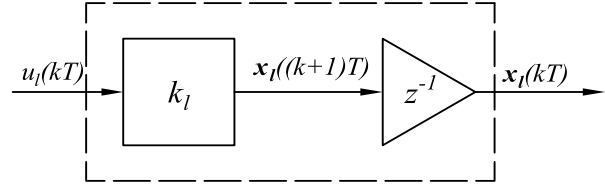


Figure 2: Blocks diagram of the camera.

The input of the camera $u_l(kT)$ is defined for the output of the inverted pendulum system, i.e. $u_l(kT) = \mathbf{C} \mathbf{x}(kT)$, according to the Figure 2 we have

$$\mathbf{x}_l((k+1)T) = k_l u_l(kT) = k_l \mathbf{C} \mathbf{x}(kT) \quad (26)$$

$$\mathbf{y}_l(kT) = \mathbf{x}_l(kT) \quad (27)$$

where

$$\mathbf{x}_l(kT) = \begin{bmatrix} p_l(kT) \\ \theta_l(kT) \end{bmatrix} \quad (28)$$

Finally the complete model of the camera-inverted pendulum system is given by

$$\mathbf{x}_c((k+1)T) = \mathbf{\Phi}_c \mathbf{x}_c(kT) + \mathbf{\Gamma}_c u(kT) \quad (29)$$

$$\mathbf{y}_c(kT) = \mathbf{C}_c \mathbf{x}_c(kT) \quad (30)$$

where

$$\mathbf{\Phi}_c = \begin{bmatrix} \mathbf{\Phi} & \mathbf{0} \\ k_l \mathbf{C} & \mathbf{0} \end{bmatrix} \quad (31)$$

$$\mathbf{\Gamma}_c = \begin{bmatrix} \mathbf{\Gamma} \\ \mathbf{0} \end{bmatrix} \quad (32)$$

$$\mathbf{C}_c = [\mathbf{0}_{2 \times 4} \quad \mathbf{I}_{2 \times 2}] \quad (33)$$

the estates vector $\mathbf{x}_c(kT)$ is defined by the estates of inverted pendulum and the estates of the camera, i.e.

$$\mathbf{x}_c(kT) = \begin{bmatrix} \mathbf{x}(kT) \\ \mathbf{x}_l(kT) \end{bmatrix}. \quad (34)$$

3 Design of a Controller and Observer

In this section it is assumed that the state of the extended system (29) - (30) is not available for measurement, and

only we have the variables that gives the camera. The control system includes linear quadratic regulator (LQR) using an estimated state vector. A control scheme is shown in Figure 3.

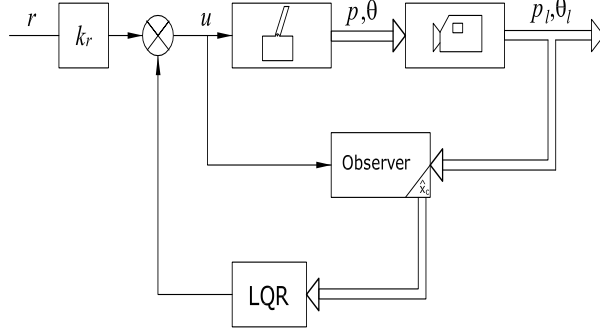


Figure 3: Control scheme with $k_r = -8.79$.

3.1 Observer Design

We proposed an observer for the extended system in the Luenberger's form.

$$\hat{\mathbf{x}}_c((k+1)T) = \Phi_c \hat{\mathbf{x}}(kT) + \Gamma_c u(kT) + \mathbf{L} \tilde{\mathbf{y}}(kT) \quad (35)$$

$$\hat{\mathbf{y}}_c(kT) = \mathbf{C}_c \hat{\mathbf{x}}(kT) \quad (36)$$

where $\hat{\mathbf{x}}$ is the estimate estate, $\tilde{\mathbf{y}}(kT)$ is the estimate error and \mathbf{L} is a gain constant matrix, with desired poles in $\lambda_1 = 0.35, \lambda_2 = 0.36, \lambda_3 = 0.37, \lambda_4 = 0.38, \lambda_5 = 0.39$ and $\lambda_6 = 0.4$, the gain matrix \mathbf{L} is obtained

$$\mathbf{L} = \begin{bmatrix} 0.0008 & 0.0 \\ 0.0338 & 0.0008 \\ 0.0 & 0.0008 \\ 0.0007 & 0.0342 \\ 0.8736 & 0.0160 \\ 0.0138 & 0.8772 \end{bmatrix}. \quad (37)$$

3.2 Controller

In this section a controller LQR is designed, where the criterion to minimize is given by

$$J = \frac{1}{2} \sum_{K=0}^{\infty} [\mathbf{x}^T(k) \mathbf{Q} \mathbf{x}(k) + u^T(k) \mathbf{R} u(k)] \quad (38)$$

where the matrices \mathbf{Q} and \mathbf{R} are

$$\mathbf{Q} = \begin{bmatrix} 80 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (39)$$

$$\mathbf{R} = 1. \quad (40)$$

And the control gain is

$$\mathbf{K} = \begin{bmatrix} -8.5878 \\ -6.8641 \\ -34.2686 \\ -6.6463 \\ 0 \\ 0 \end{bmatrix}^T. \quad (41)$$

4 Simulation Results

The simulations of system in close-loop were performance in Matlab [10]. The simulation results are given in Figure 4

5 Conclusions

In this work the control of the inverted pendulum is presented using a camera of vision as a sensor to observe the position of the pendulum and the position of the cart. The simulation results show a good performance of the close-loop system.

Currently, we are working in real-time implementation using Texas Instrument DSP.

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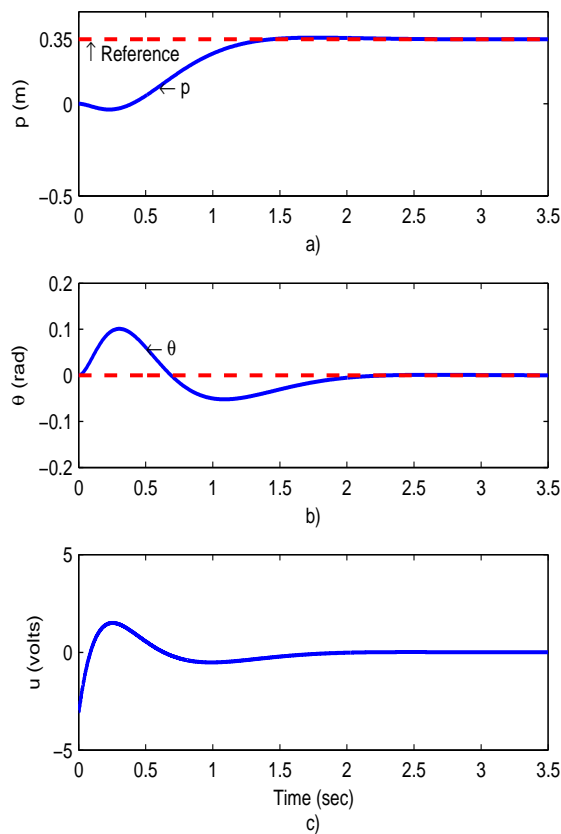


Figure 4: Simulation results of the system inverted pendulum for an step input reference.

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